Homework 12

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Due April 27; the last problem set.

1. Check in detail the relationships

$$E[(q_j(0) - \frac{\gamma_j x(0)}{f_j^2})(q_k(0) - \frac{\gamma_k x(0)}{f_k^2)}] = \delta_{jk}\frac{T}{f_j f_k},$$

$$E[p_j(0)p_k(0)] = T\delta_{jk},$$

$$E[F_n(t)F_n(t-t')] = -TK_n(t')$$

where

$$F_n(t) = \sum_j \gamma_j(q_j(0) - \frac{\gamma_j x(0)}{f_j^2})\cos(f_j t) + \sum_j \gamma_j p_j(0)\frac{\sin(f_j t)}{f_j}$$

at the end of section 6.2.

2. Consider the ordinary differential equation $d\phi(x,t)/dt = 1, \phi(x,0) = x$, construct the corresponding Liouville equation, solve this Liouville equation explicitly when the initial datum is $u(x,0) = x$, and verify that $u(x,t) = \phi(x,t)$.

3. Check in detail the derivation of the approximation of the Hald model in section 6.6.

4. (For extra credit): Determine $\Phi_{1,2}(t)$ for the previous problem for $0 \leq t \leq 2$ in the following two ways: 1. Approximately by solving the approximating two-by-two system of ODE's , and 2. By repeatedly sampling $x_3, x4$ given x_1, x_2, solving the four-by-four Hald system and averaging. Compare the two solutions.

Hints: To sample the canonical density use Markov chain Monte-Carlo; to solve the ordinary differential equations the routines in Matlab will do fine.