Comparison of Model Order Reduction Techniques on High-Fidelity Electrical, Mechanical, and Biological Systems

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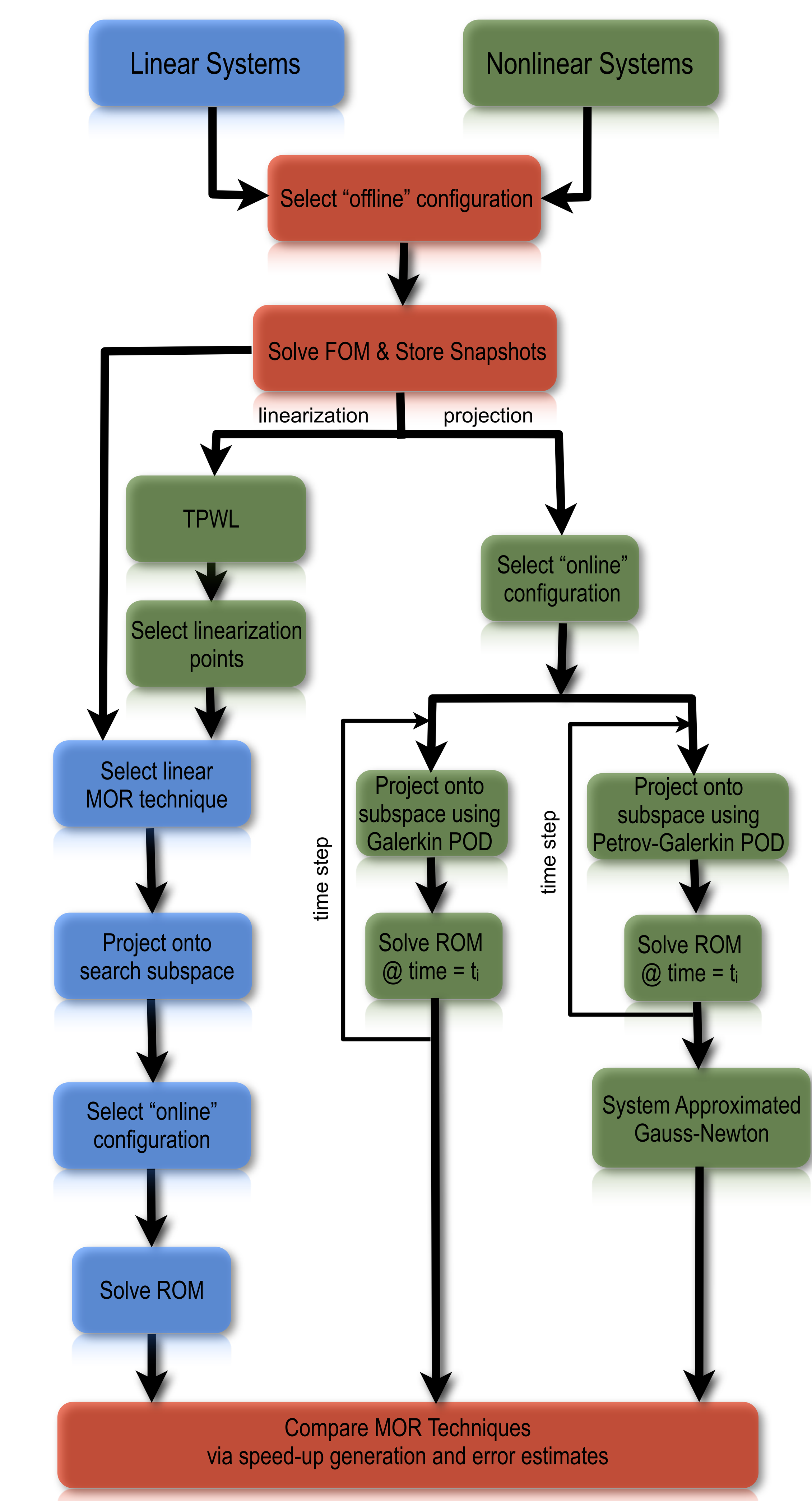
Motivation

There has been little work done comparing model reduction techniques across a variety of linear and nonlinear systems; the goal of this work is to provide a testbed that enables a thorough comparison of techniques on different types of systems.

Model reduction of simple systems such as linear time-invariant systems is relatively mature, while nonlinear model reduction is much more complicated.

A natural question is: which of these model reduction techniques performs best on which problems?

Analysis Strategy



Further Information

The first author can be contacted at bokie89@sbcglobal.net (Matthew J. Zahr) for additional information regarding this project.

Experiment

Linear Module

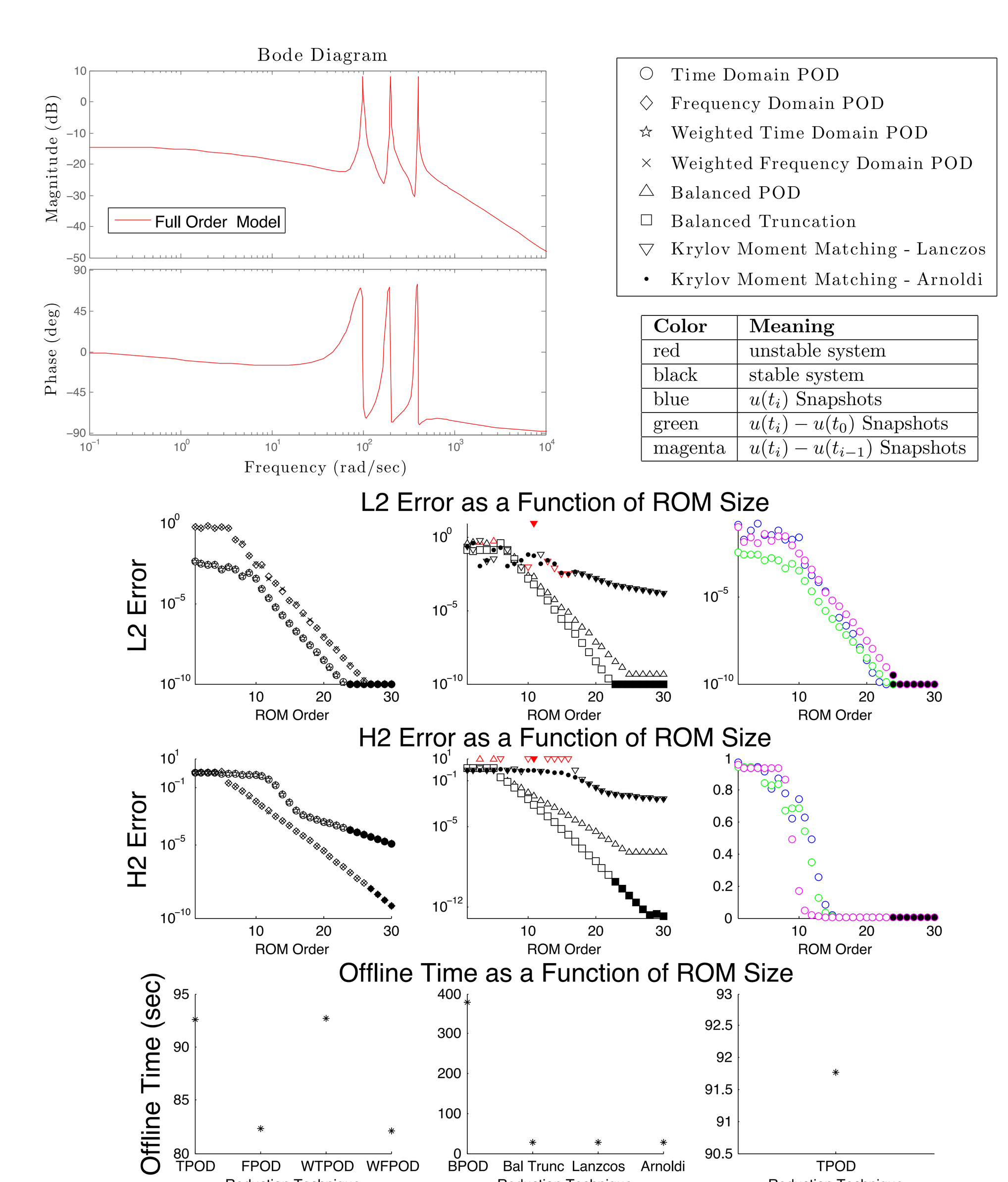
- Systems
  - Multiple Mass-Spring-Damper
  - "Penzl" Problem [5]
  - Heat Flow
- MOR Techniques
  - POD [1,2]
    1. time domain
    2. frequency domain
  - weighted POD [7]
    3. time domain
    4. frequency domain
  5. Balanced POD [1,2]
  6. Balanced Truncation [1,2]
  - Krylov Moment Matching [2]
    7. Two-Sided Lanczos
    8. Arnoldi

Nonlinear Module

- Systems
  - 1D Shock Propagation [6]
  - Transmission Line [6]
  - Neuron Activation Model [4]
  - Steady-State Problem [4]
  - MEMS [6]
- MOR Techniques
  1. Galerkin Projection
  2. Least-Squares Petrov-Galerkin Projection (LSPG) [3]
  3. System Approximated Gauss-Newton (SAGN) [3]
  4. Trajectory Piecewise Linear Approximation (TPWL) [6]

Linear Module Results

- The only MOR technique with unstable ROMs is the Lanczos Krylov method.
- The Krylov methods are the least expensive methods, but induce the largest error. The time POD methods are the most expensive.
- Balanced Truncation has low cost, low error in both norms, and remains stable throughout all ROMs investigated.
- The snapshots referencing the initial condition have the lowest error and those referencing the previous condition have the largest error.



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Nonlinear Module Results

All of the nonlinear MOR techniques are heavily dependent on adjustable parameters. Before we can compare the MOR techniques against each other, it is necessary to determine the optimal parameters for each MOR technique.

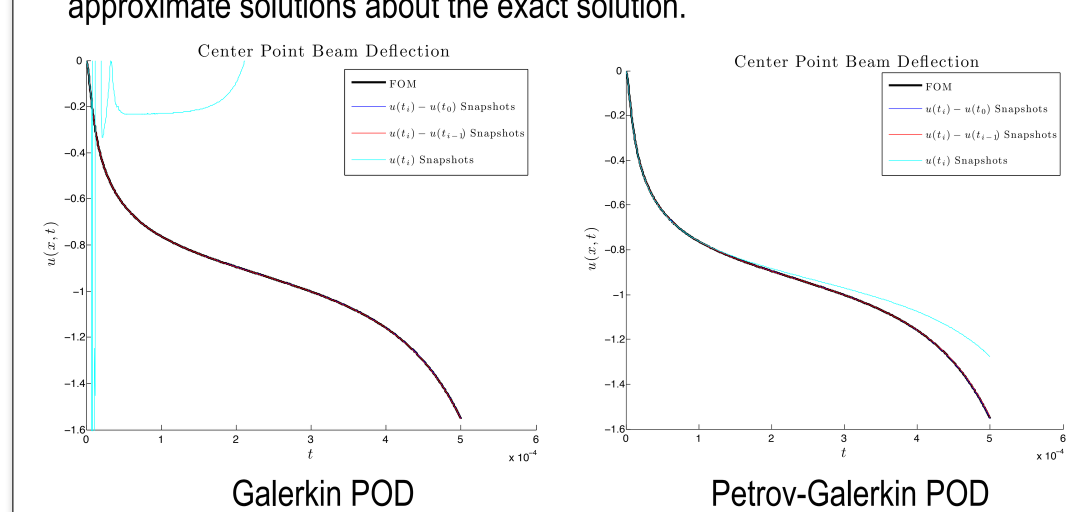
This section presents the following:

- POD Snapshot Selection Comparison
- TPWL Linearization Pt. Selection Algorithm Comparison
- SAGN Parameter Comparison
- Robustness Analysis (different training and online inputs)
- Inter-Method Comparison

POD Snapshot Selection Comparison

The three snapshot collections compared in the section are: 1) state vector referencing the initial condition, 2) state vector referencing the previous time step, 3) unreferenced state vector. These collections were compared for both the Galerkin (same left and right projection bases) and Petrov-Galerkin (different left and right projection bases) POD techniques.

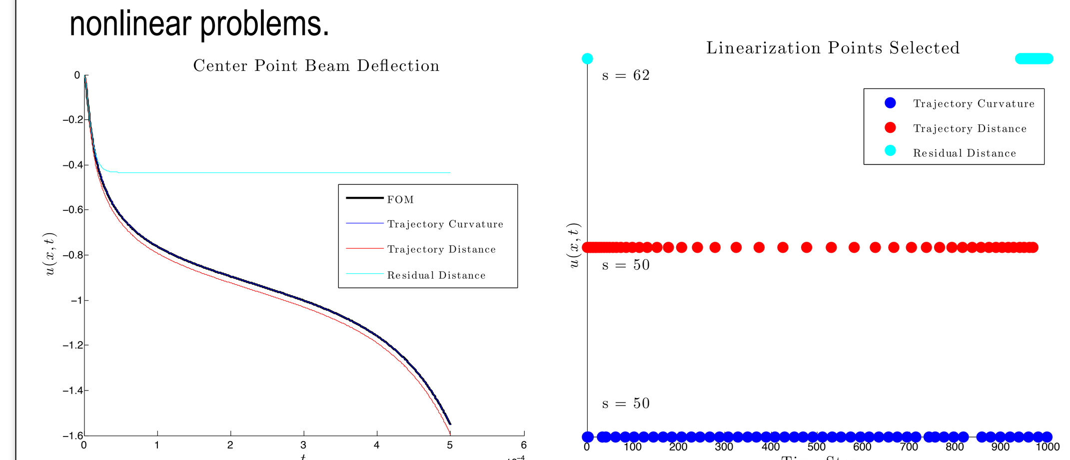
For both Galerkin and Petrov-Galerkin POD projections, the snapshots 1 are best with respect to error minimization and the snapshots 3 led to the worst results. Furthermore, LSPG projection is more stable than Galerkin projection. The stability is assessed by magnitude of the oscillations of the approximate solutions about the exact solution.



TPWL Linearization Pt. Selection Algorithm Comparison

TPWL is a method that reduces the computational expense of nonlinear systems by linearizing the system about points along the solution trajectory. The selection of linearization points is vital to the accuracy of TPWL solutions.

The algorithms considered for selection points are classified as follows: 1) Trajectory Curvature (developed by first author), 2)Trajectory Distance, 3) Residual Distance. The first method performs best for this problem and the third method performs the worst. This result did not generalize to the other nonlinear problems.



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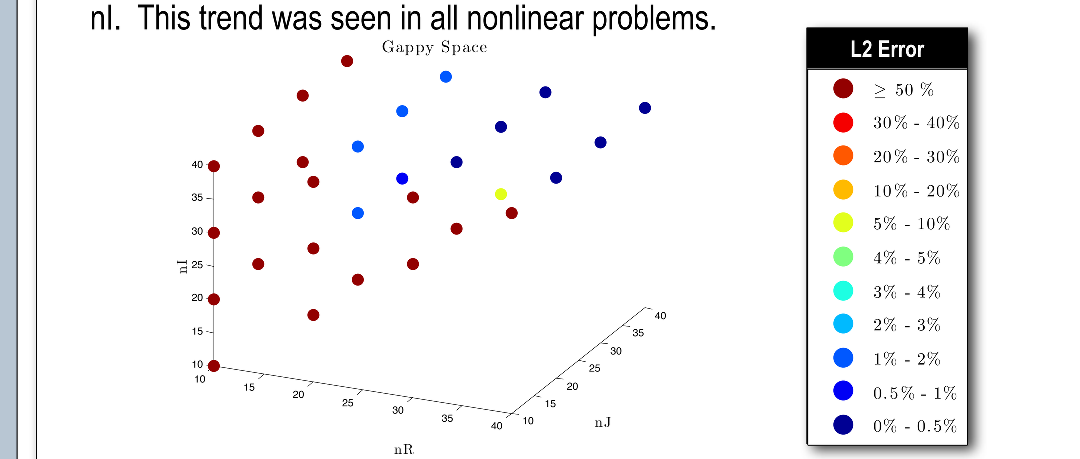
Nonlinear Module Results (continued)

SAGN Parameter Comparison

SAGN is a technique for reducing the computational cost of nonlinear problems that was developed by Carlberg & Farhat [3]. There are three vital SAGN parameters: nR = number of POD basis vector to represent the residual, nJ = number of POD basis vectors to represent the Jacobian, and nI = number of indices for which the Jacobian and Residual are computed.

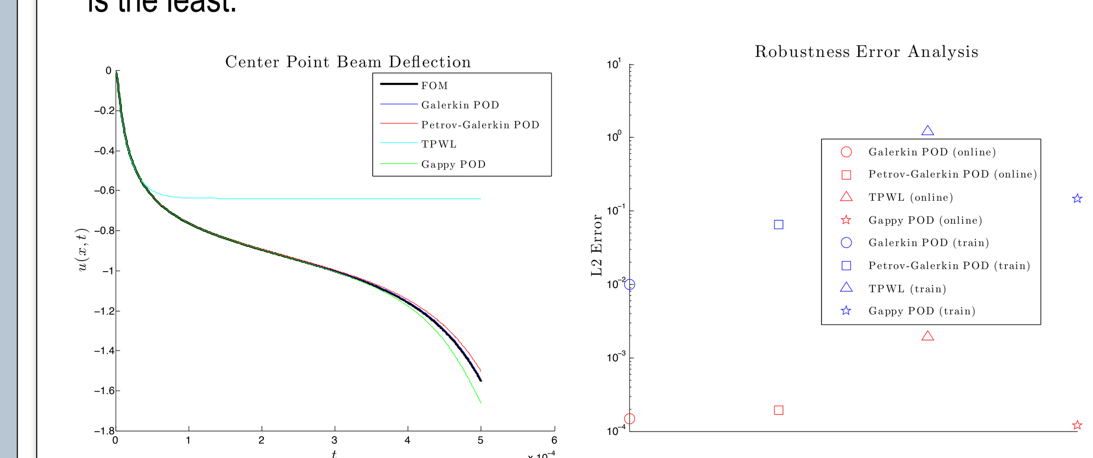
The error is strongly dependent on nR and nJ, but only weakly dependent on nI. This trend was seen in all nonlinear problems.



Robustness Analysis

In this section, unlike the other sections, the online and offline (training) inputs differ. Train input = 49H(t) ; Online input = 81H(t).

All of the POD methods have a high degree of robustness, while TPWL does not. Galerkin POD is the most robust of the POD methods and SAGN is the least.

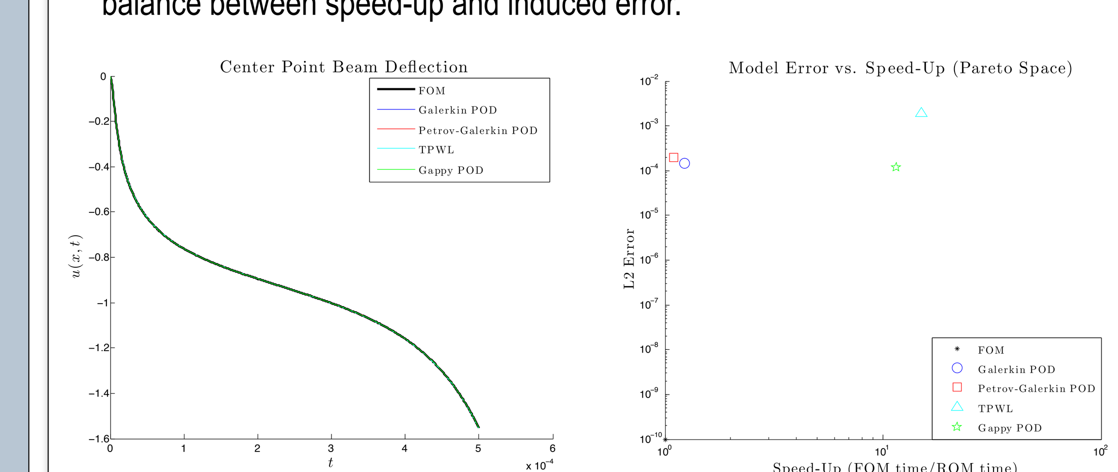


Inter-Method Comparison

With the optimal parameters gathered from the previous four sections, a final comparison is made to determine the best MOR technique for this problem.

TPWL and Gappy POD have the largest induced error and the most significant speed-up. The other POD methods have negligible speed-up and negligible induced error. These results generalized to all nonlinear problems.

Conclusion: SAGN performs best for the MEMS problem due to favorable balance between speed-up and induced error.



Conclusion

The MOR testbed provides a means to effectively compare MOR techniques for linear and nonlinear problems. The testbed was useful for comparing MOR techniques and is intended for researchers to test new methods against those currently in existence.

For the Penzl problem, Balanced Truncation performed the best. This observation roughly generalized to the other linear problems. For the MEMS switch, SAGN performed best, which is an observation that did not generalize to all nonlinear problems.