A Hyperreduced FE^2 Method for Real-Time Multiscale Simulations

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This project introduces an efficient numerical method for modeling and solving structural multiscale problems that inevitably arise in many fields, including orthopedics. The method leverages state-of-the-art reduced-order models, built on-the-fly, at the various levels of the multiscale simulation to reduce the cost well below that of a standard single-scale simulation. A speedup of four orders of magnitude was observed on a representative model problem.

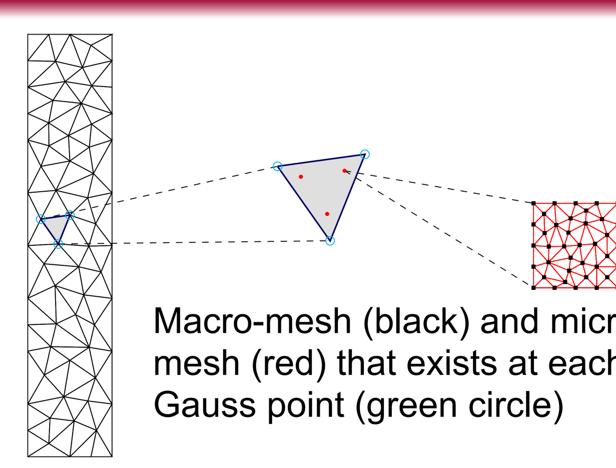
Motivation

Fast and reliable orthopedic modeling is of utmost interest to the US Military as the primary factor that drives the design of humvees is the health and safety of its occupants. We turn to computational modeling due to the expense and difficulty associated with experimentation in this scenario.

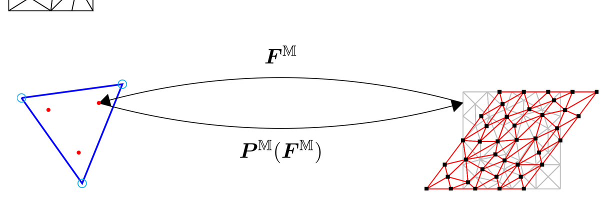
- Orthopedic modeling is difficult due to the multiple length scales that comprise bone and accepted material laws poorly model bone porosity
- Infeasible to use finite element method with homogenous mesh
- FEM meshes for each scale with “scale-bridging” methods shown promise, but still expensive as billions of queries to lower scales may be required
- Introduce fast, accurate reduced-order model at various scales for *game-changing* speedups

Multiscale Modeling via FEM^2

- Two scales: the macroscale and microscale
- A mesh of the macroscale is used to discretize the geometry *without* attempting to resolve the fine-scale features
- Due to difficulty of defining an analytical material law that accurately models the fine-scale features, a *microscale* finite element model is employed
- Given the macroscale displacements ($\boldsymbol{u}^{\mathbb{M}}$), the deformation gradient ($\boldsymbol{F}^{\mathbb{M}}$) is computed at each gauss point and used to define boundary conditions for microscale model
- Homogenization of the microscale stresses leads to the pointwise macroscale stress $\boldsymbol{P}^{\mathbb{M}}(\boldsymbol{F}^{\mathbb{M}})$
- This implicitly assumes a *separation of scales*



Macro-mesh (black) and micro-mesh (red) that exists at each Gauss point (green circle)



Transmission of deformation gradient from macro to micro and homogenized stress from micro to macro

Hierarchy of Reduced-Order Multiscale Models

Model Reduction at Microscale

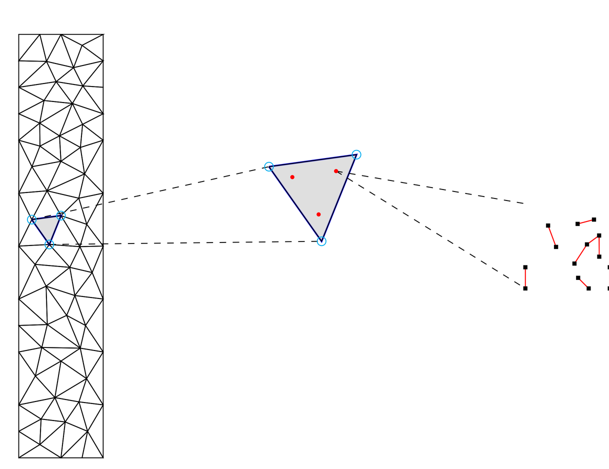
- Let $\boldsymbol{u}^{\mu}$ be the microscale displacements and assume it can be well-approximated in a low-dimensional subspace

$$\boldsymbol{u}^{\mu} \approx \boldsymbol{\Phi}^{\mu}\boldsymbol{y}$$

- For general nonlinear problems, the state reduction is not sufficient to realize noteworthy speedups. An additional step, called hyperreduction, is required that effectively sparsifies the mesh.
- The Energy Conserving Sampling and Weighing (ECSW) is employed in this work and can interpreted as an approximate quadrature rule for evaluating the internal forces

$$\int_{\Omega} f(\boldsymbol{x})\, d\boldsymbol{x} \approx \sum_{e \in \mathcal{E}} \int_{\Omega_e} f(\boldsymbol{x})\, d\boldsymbol{x} \approx \sum_{e \in \mathcal{E}' \subset \mathcal{E}} \int_{\Omega_e} \alpha_e f(\boldsymbol{x})\, d\boldsymbol{x}$$

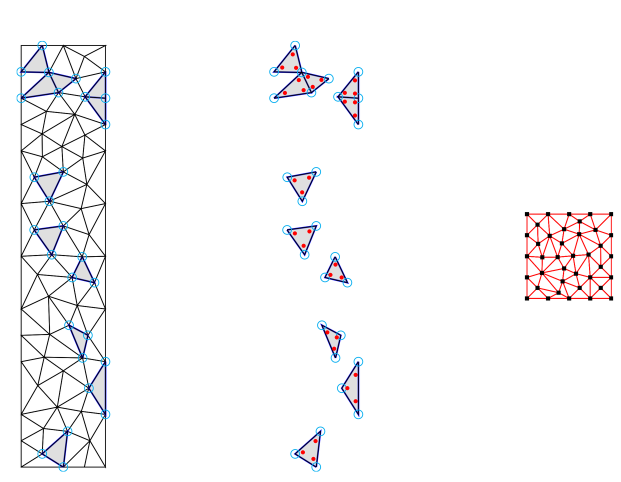
where $\mathcal{E}, \mathcal{E}'$ are the full and sparsified finite element mesh, respectively.



FEMxROM or ROM^2: Microscale simulation replaced with hyperreduced model (ECSW)

On-the-Fly Training

- The reduced-order basis ($\boldsymbol{\Phi}^{\mu}$) is built by compressing solutions snapshots using Proper Orthogonal Decomposition.
- A small, representative set of elements are extracted from the macro-mesh and used to generate solution snapshots by solving for the microscale displacements at its quadrature points.
- The representative set of elements can be selected heuristically or using automated algorithms such as clustering or greedy sampling.



Training Phase*: Select representative* elements, solve full-scale micro-model and store snapshots, compress snapshots and build sample mesh

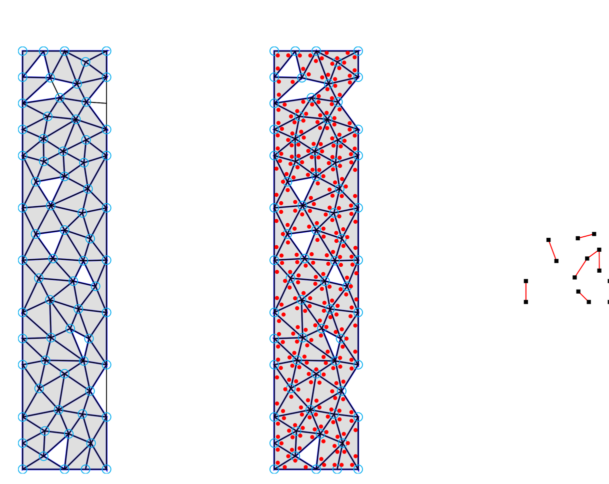
Model Reduction at Macroscale

- Further reduction in computational cost can be achieved by introducing a reduced-order model, along with mesh sparsification, at the macroscale

$$\boldsymbol{u}^{\mathbb{M}} \approx \boldsymbol{\Phi}^{\mathbb{M}}\boldsymbol{y}^{\mathbb{M}}$$

Summary

- We have reduced the nested finite element method (FEM^2) for multiscale modeling to a nested reduced-order method (ROM^2) by introducing a reduced-order model and sample mesh at both scales.

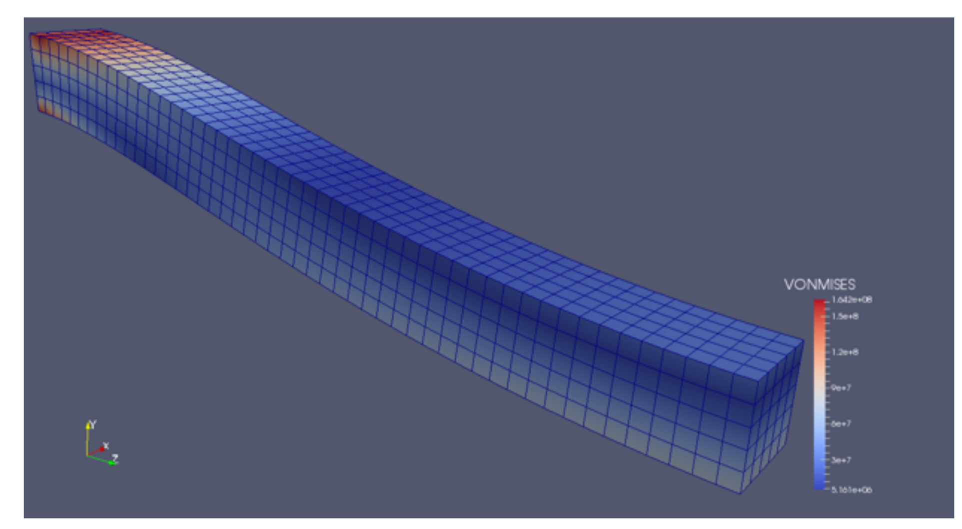


Online Phase: At all elements not used in training, solve reduced-order micro-model and communicate homogenized stress to macroscale

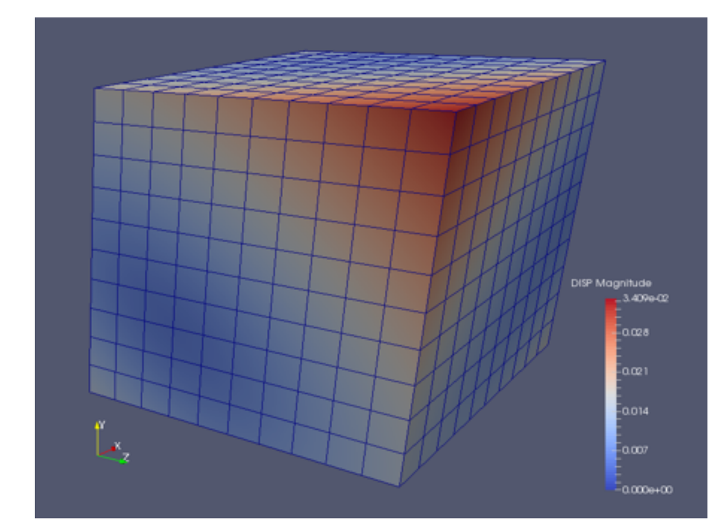
Application to Model Problem

The proposed multi multiscale ROM method has been implemented in the AERO-S, a massively-parallel finite element implementation that solves elliptic partial differential equations. The method is demonstrated on two model problems. The macro-mesh and micro-mesh are shown below and the simulation setup is summarized in the table.

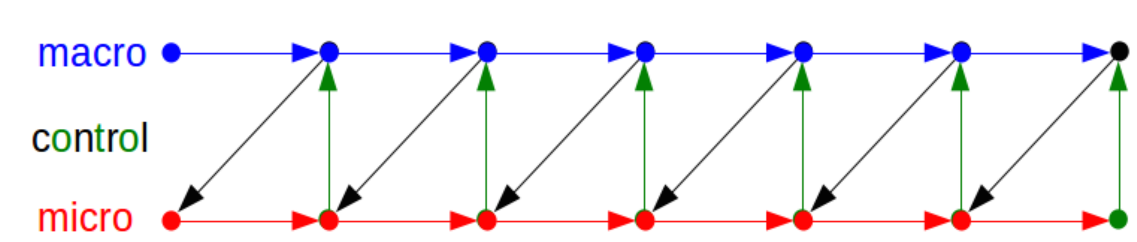
Macro-mesh



Micro-mesh

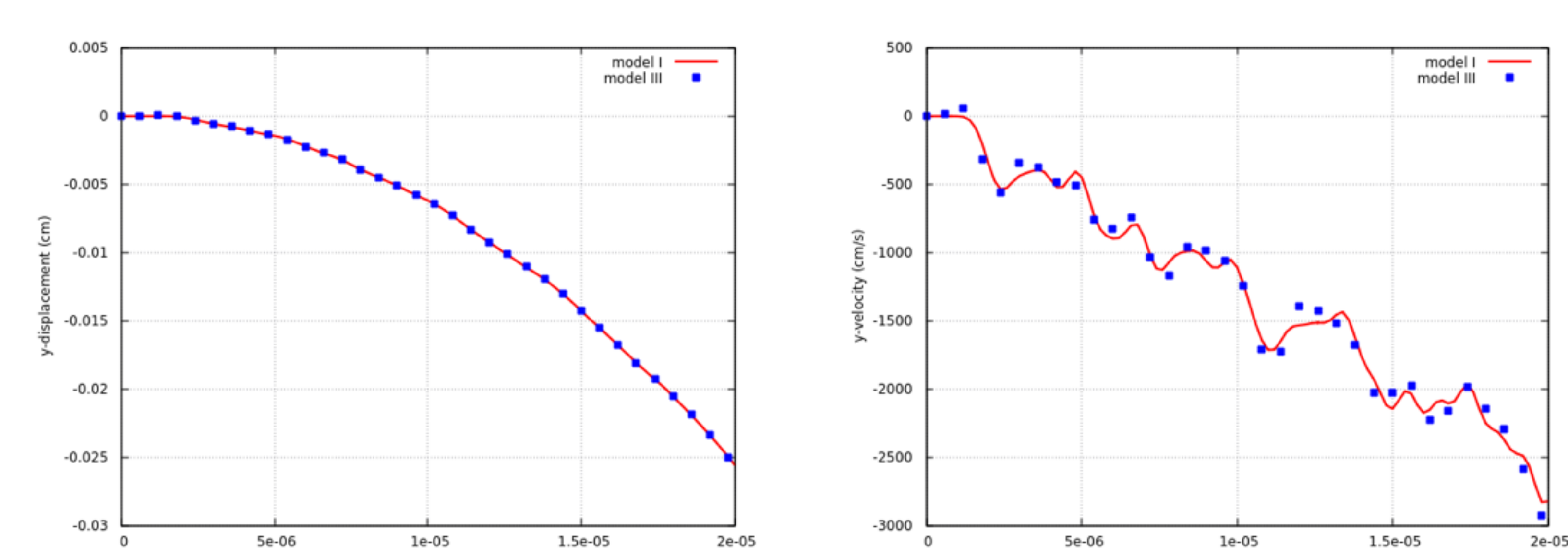


The implementation in AERO-S employs the below workflow whereby the macro-mesh evaluates the deformation gradient at quadrature nodes (blue arrows), a “control law” is used to communicate this information to the micro-mesh (black arrows) and used to define boundary conditions, and the micro model is solved and the homogenized stress communicated to the macro-mesh (green arrows).



Multiscale workflow implementation in AERO-S

The high-quality of the multiscale ROM solution is demonstrated below by comparing the y-displacement and velocity at a sensor in the mesh. The displacements predicted by the reduced-order model are indistinguishable from the full-scale model and only minor variations in the velocity are observed. A timing comparison in the below table shows the reduced-order model framework is 4 orders of magnitude faster than the full-scale one.

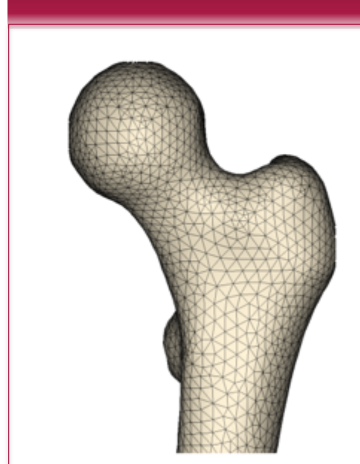


Comparison of HDM multiscale simulation to two-level ROM multiscale simulation in terms of y-displacement (left) and y-velocity (right) at sensor

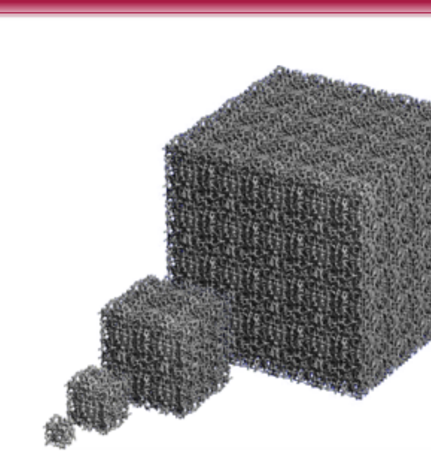
	# macro elements	# macro dof	# micro elements	#micro dof	# CPU	wall time
FEM^2	1250	5508	1000	3993	80	1.32×10^4 sec
ROM^2	40	12	8	9	40	3.27 sec

Summary of FEM^2 and ROM^2 setup and timing results. ROM^2 is 4 orders of magnitude faster with minimal error introduced. Simple a-priori training used for ROM; future work will include on-the-fly training.

Conclusion



- New method introduced for solving structural multiscale problems that leverages state-of-the-art model reduction technology
- Potentially *game-changing* speedups demonstrated on model problems
- Future work will apply the method to real bone geometry under loads expected in under-body blast scenario



Collaborations

The following collaboration efforts are planned:

Richard Becker, ARL fellow and engineer in ARL’s Weapons and Materials Research Directorate, who originally brought the problem to our attention.