Rapid Topology Optimization using Reduced-Order Models

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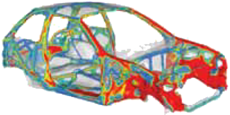
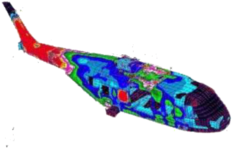
Motivation

- For industry-scale design problems, topology optimization is a beneficial tool that is *time and resource intensive*
  - Large number of calls to structural solver usually required
  - Each structural call is expensive, especially for nonlinear 3D High-Dimensional Models (HDM)
- Use a Reduced-Order Model (ROM) as a surrogate for the structural model in a material topology optimization loop
  - Large speedups over HDM realized





0-1 Material Topology Optimization

$$\begin{aligned} \underset{\boldsymbol{\chi} \in \mathbb{R}^{n_{el}}}{\text{minimize}} \quad & \mathcal{L}(\mathbf{u}(\boldsymbol{\chi}), \boldsymbol{\chi}) \\ \text{subject to} \quad & \mathbf{c}(\mathbf{u}(\boldsymbol{\chi}), \boldsymbol{\chi}) \leq 0 \end{aligned}$$

- $\mathbf{u}$ is implicitly defined as a function of $\boldsymbol{\chi}$ through the HDM equation

$$\mathbf{f}^{int}(\mathbf{u}) = \mathbf{f}^{ext}$$

$$\mathbb{C}^e = \mathbb{C}_0^e \boldsymbol{\chi}_e \qquad \rho^e = \rho_0^e \boldsymbol{\chi}_e \qquad \boldsymbol{\chi}_e = \begin{cases} 0, & e \notin \Omega^* \\ 1, & e \in \Omega^* \end{cases}$$

Reduced-Order Model

- Model Order Reduction (MOR) assumption
  - State vector lies in low-dimensional subspace defined by a Reduced-Order Basis (ROB) $\boldsymbol{\Phi} \in \mathbb{R}^{N \times k_{\mathbf{u}}}$

$$\mathbf{u} \approx \boldsymbol{\Phi} \mathbf{y}$$

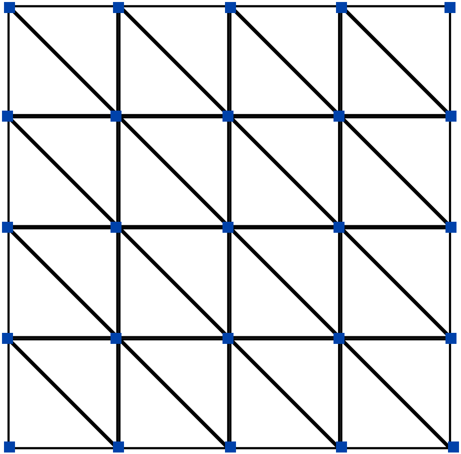
  - $k_{\mathbf{u}} \ll N$
- N equations, $k_{\mathbf{u}}$ unknowns

$$\mathbf{f}^{int}(\boldsymbol{\Phi} \mathbf{y}) = \mathbf{f}^{ext}$$

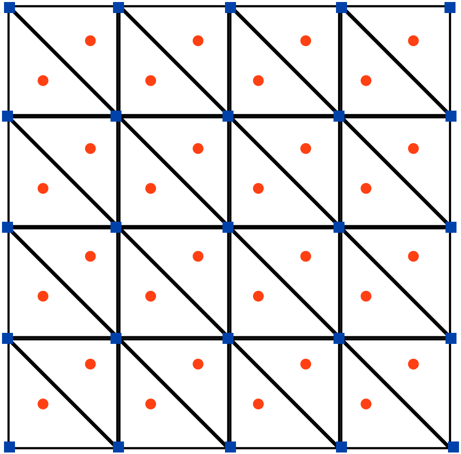
- Galerkin projection

$$\boldsymbol{\Phi}^T \mathbf{f}^{int}(\boldsymbol{\Phi} \mathbf{y}) = \boldsymbol{\Phi}^T \mathbf{f}^{ext}$$

Model and Optimization Variable Reduction

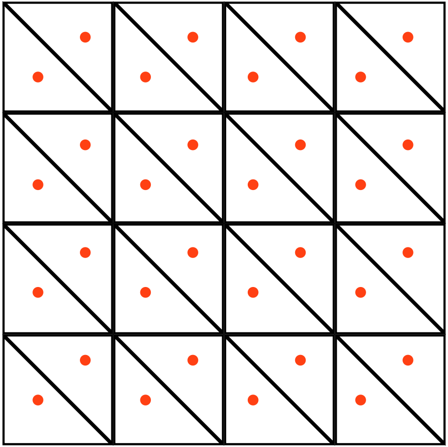


Model and Optimization Variable Reduction



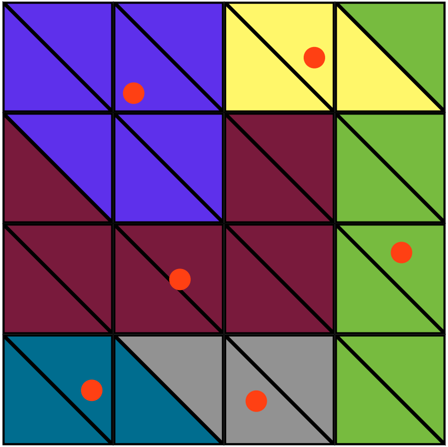
Model and Optimization Variable Reduction

- Model reduction tells us how to reduce $\mathbf{u}$, $\mathbf{f}^{\text{int}}$, $\mathbf{f}^{\text{ext}}$
- How to reduce χ?



Model and Optimization Variable Reduction

- Model reduction tells us how to reduce $\mathbf{u}$, $\mathbf{f}^{\text{int}}$, $\mathbf{f}^{\text{ext}}$
- How to reduce $\boldsymbol{\chi}$?



Reduced Topology Optimization

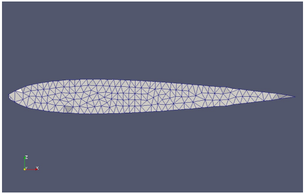
$$\begin{aligned} \underset{\boldsymbol{\alpha}_r \in \mathbb{R}^{n_\alpha}}{\text{minimize}} \quad & \mathcal{L}(\mathbf{y}(\boldsymbol{\alpha}_r), \boldsymbol{\alpha}_r) \\ \text{subject to} \quad & \mathbf{c}(\mathbf{y}(\boldsymbol{\alpha}_r), \boldsymbol{\alpha}_r) \leq 0 \end{aligned}$$

- $\mathbf{y}$ is implicitly defined as a function of $\boldsymbol{\alpha}_r$ through the ROM equation

$$\boldsymbol{\Phi}^T \mathbf{f}^{int}(\boldsymbol{\Phi}\mathbf{y}) = \boldsymbol{\Phi}^T \mathbf{f}^{ext}$$

Problem Setup

- St. Venant-Kirchhoff
- 90,799 tetrahedral elements
- 86,493 dof
- Static simulation with load applied in 10 increments
- Loads: Bending (X- and Y- axis), Twisting, Self-Weight
- ROM size: $k_{\mathbf{u}} = 5$



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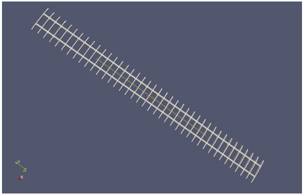
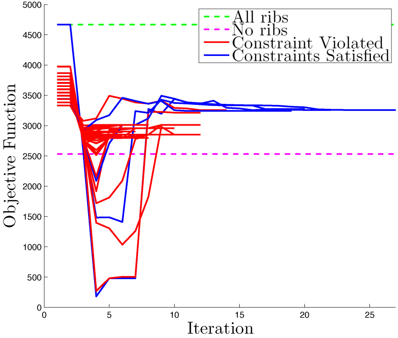


40 Ribs

Optimization Results

Minimize structural weight subject to displacement constraints



Optimization Iterates

	Offline (s)	Online (s)	Speedup	Error (%)
HDM	-	811	-	-
ROM	9,603	1.51	538	1.73